Elementary maths for GMT

Calculus

Part 2.2: Function analysis

Function analysis

- For analyzing and drawing a given function f(x)
  - Find zero crossings ($f(x) = 0$)
  - Find zero crossings of the derivative (extrema) *[and sometimes of higher order derivatives]*, look at sign around them
  - Look at behavior for $x \to \pm\infty$ (or at domain ends)
  - Look at singularities (where the function is undefined, typically from a division by zero)
  - Draw some likely points

Example

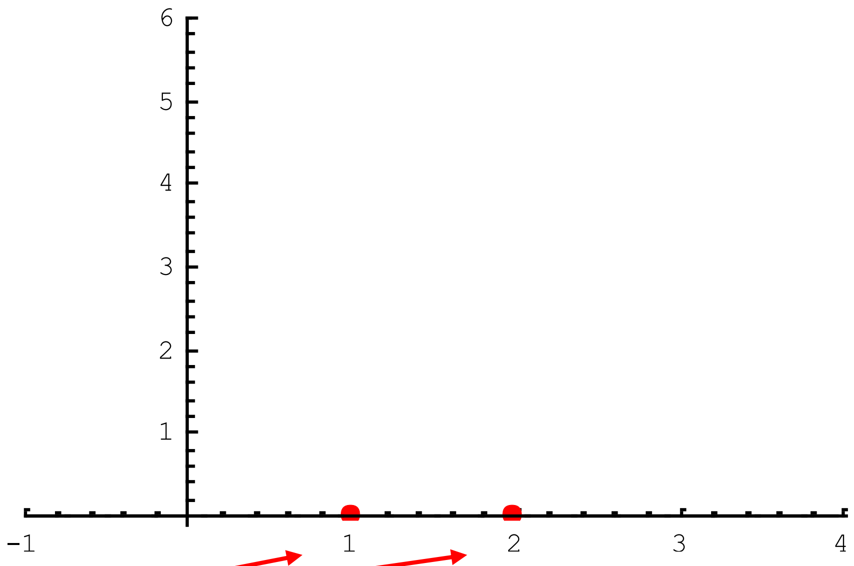
- The function $f(x) = x^2 - 3x + 2$
  - function zero crossings

$$f(x) = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

at $x = 1 \vee x = 2$



Example

- The function $f(x) = x^2 - 3x + 2$

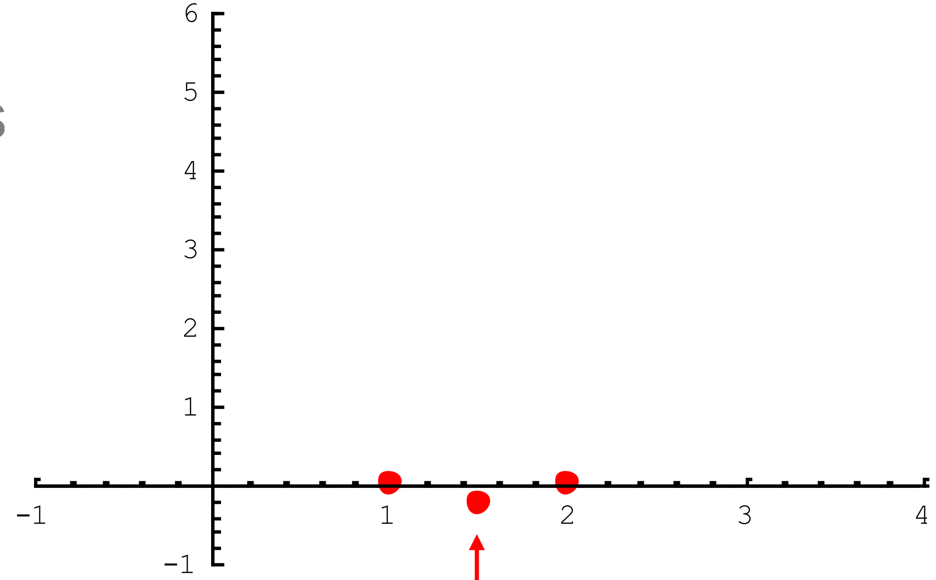
  – derivative zero crossings

  $f'(x) = 2x - 3 = 0$

  at $x = 3/2$

  – sign

$$\begin{array}{l|c|c} & \multicolumn{2}{c}{\tfrac{3}{2}} \\ f'(x) & < 0 & > 0 \\ & \searrow & \nearrow \end{array}$$

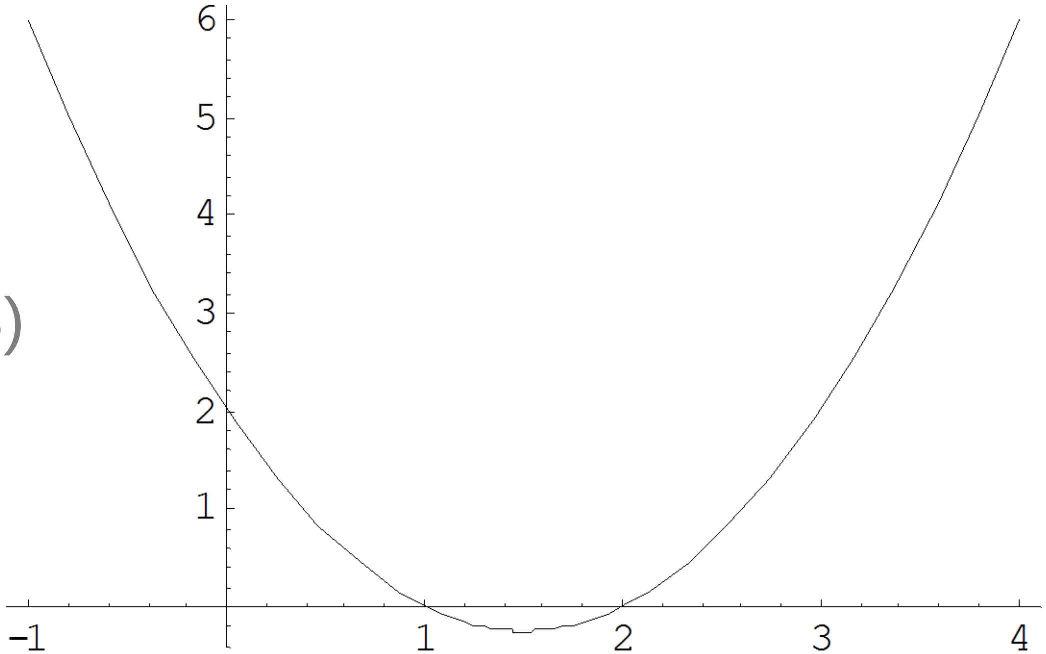


minimum at

$$\left(\frac{3}{2}, f(\frac{3}{2})\right) = \left(\frac{3}{2}, -\frac{1}{4}\right)$$

Example

- The function $f(x) = x^2 - 3x + 2$
  - when $x \to \pm\infty$, $f(x) \to +\infty$
  - no singularity
  - draw some points (e.g. for x = 0 and 3)



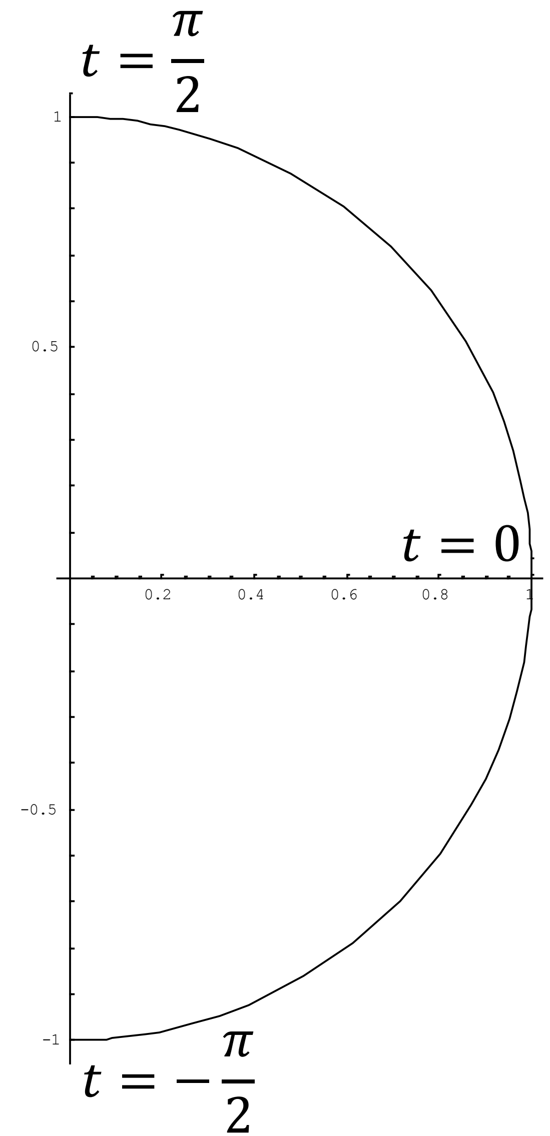
Parametric curves

- Of the form $$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$
- Analysis similar to explicit functions
  - Find zero crossings of component functions
  - Look at derivative (= tangent vector!)
  - Look at behavior for $t \to \pm\infty$ (or at domain ends)
  - Look at singularities
  - Draw some likely points

Example

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \text{ for } -\tfrac{\pi}{2} \le t \le \tfrac{\pi}{2}$$

- Examine $x(t)=0$, $y(t)=0$
- Examine $t = \pm\frac{\pi}{2}$
- No singularity



Example

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \text{ for } -\tfrac{\pi}{2} \le t \le \tfrac{\pi}{2}$$

derivative:

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

example for $t = 0$:

tangent vector $\begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$